Demand Response Design based on a Stackelberg Game in Smart Grid

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Abstract—The future power grid will incorporate the information and communications technology (ICT), that is, smart grid. With help of a two-way communication infrastructure, a real-time demand response can be applied. A smart grid network consisting of one retailer and many customers is considered. The retailer broadcasts the electricity price information. Then, the customers consume electricity to maximize their payoffs. This paper designs a Stackelberg game and derives the solution of the game, i.e., a Stackelberg equilibrium.

I. INTRODUCTION

Current power grid is not enough stable to supply the electricity continuously. One important trend in the power grid is applying information and communication technology (ICT) to make the power grid more robust, that is, smart grid [1].

Since electricity is not a storable resource, the power grid always keeps the supply and the demand equally. A traditional method to keep the balance is controlling the supply part to the estimated future demand. In the smart grid system, however, different approach is considered, that is, controlling the demand part with help of a real-time two-way communication system. This is called demand-side management (DSM) [2].

Among DSM, demand response (DR) [3] is an indirect way to control the demand through hourly pricing information. Then, smart customers can efficiently reduce their electrical charges by shifting its deferrable load to the low price hours. Common programs in this group are critical-peak pricing (CPP), time-of-use pricing (TOU), and real time pricing (RTP).

There is a conflict of interests between a retailer and customers to maximize their payoffs. This is a game situation with different information. The retailer can choose its price before the customers choose their electricity consumption level, that is, the retailer and customers are leader and followers, respectively. We design this problem as a Stackelberg game and derive its solution in this paper.

II. SYSTEM MODEL

We consider several customers that are served by a single retailer as shown in Fig. 1. The retailer participates the whole-sale market to buy electricity and it resales the electricity to the customers. It is assumed that the retailer is connected with the customers through both power and communication networks.
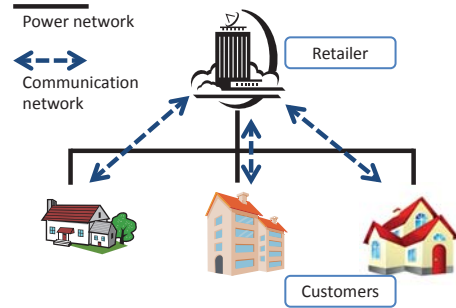
Fig. 1. Smart grid system model.

The solid and dotted lines represent a power distribution and communication networks, respectively. Several communication candidates are available for the smart grid system such as Wi-Fi, WiMAX, and smart utility network (SUN) [4]. The retailer announces its unit price of the electricity p and then each customer i decides its demand x_i according to the p.

A. Electricity Demand Model for Customers

We classify the appliances into three types. First type is price inelastic appliances such as light and refrigerator. Since this type appliances do not change their electricity consumption level, we do not consider them in this paper.

Second type is battery type elastic appliances. We call it type 1 appliance. These appliances want to consume electricity as much as possible. We model this type's utility function of customer i as

$$U_{i1}(x_{i1}, w_{i1}) = w_{i1}\log(1 + x_{i1}), \qquad (1)$$

where x_{i1} and w_{i1} are the electrical consumption level in kWh and weight to the type 1 appliance of customer i, respectively. A customer who wants to consume more electricity has higher w_{i1} and vise versa.

Third type (type 2 appliance) can be modeled with a convex disutility function. That is, according to the electricity consumption level, the customer's degree of satisfaction varies. Heating, ventilating, and air conditioning (HVAC) system is an example of this type appliance. The utility function of the type 2 application of customer i is defined as

$$U_{i2}(x_{i2}, w_{i2}) = -w_{i2}(x_{ic} - x_{i2})^2, \qquad (2)$$

where x_{ic}, x_{i2}, and w_{i2} are the most convenient electricity consumption level, the electrical consumption level in kWh and weight to the type 2 appliance of customer i, respectively.

The weight in this type represents how severely the customer feels with $|x_{ic} - x_{i2}|$.

III. STACKELBERG GAME AND ITS SOLUTION

We design a Stackelberg game where there are one retailer [leader] and many customers [followers]. At first, the leader selects p to maximize its profit with the knowledge of the customers behavior. Then, followers choose their electricity consumption level given p. This is a backward induction technique to derive equilibrium of the Stackelberg game. The solution of the optimization problem is, therefore, the equilibrium (i.e., subgame perfect equilibrium) of the game.

A. Customer Side Analysis

Each customer chooses its electricity consumption level to maximize its payoff with the electricity price p which is broadcasted by the retailer. It is assumed that the weight of each customer is predetermined. We define the payoff function of customer i as

$$g_i(x_i) = U_{i1}(x_{i1}, w_{i1}) + U_{i2}(x_{i2}, w_{i2}) - p(x_{i1} + x_{i2}), \quad (3)$$

where x_i is the amount of consumed electricity of customer i. This function is a convex function since a summation of convex functions is also a convex function. Since all the three terms are differentiable, the payoff function is also differentiable.

B. Retailer Side Analysis

The retailer determines the electricity price p to maximize its profit with the knowledge of each customer's action. We define the payoff function $f(\cdot)$ of the retailer as

$$f(p) = p \sum x_i(p) \quad (4)$$

where $x_i(p)$ is the amount of consumed electricity for customer i with price p.

We assume that w_{i1}, w_{i2}, and x_{ic} for every i are known to the retailer. Thus, from the viewpoint of the retailer, we can make an optimization problem of customers' behavior as

$$(\mathbf{P}) \quad \max_{\mathbf{x}} \quad \sum_{i \in \mathcal{N}} g_i(x_i)$$
$$\text{subject to} \quad x_{i1}, x_{i2} \geq 0 \text{ for all } i \in \mathcal{N}$$
$$\sum_{i \in \mathcal{N}} x_i \leq X_{\max},$$

where $\mathbf{x}$ and $X_{\max}$ are a set of x_i for all $i \in \mathcal{N}$ and maximum electricity capacity for the retailer, respectively.

The objective function is a summation of convex functions, so it is a concave function. The three constraints are linear. The problem $(\mathbf{P})$, therefore, is a convex optimization problem. It is supposed that there is feasible $\mathbf{x}$, i.e., the inequality constraint holds with strict inequality. This is the Slater's condition, that is, one constraint qualification of strong duality [5]. Through a technique to solve the convex optimization problem, we can obtain the solution $\mathbf{x}^*$ and p of the game as

$$x_{i1}^* = \begin{cases} \frac{w_{i1}}{p} - 1, & \text{if } w_{i1} \geq p, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

TABLE I
CONSUMED ELECTRICITY IN UNITS OF KILOWATT HOURS

	x_{11}	x_{12}	x_{21}	x_{22}	x_{31}	x_{32}	x_{41}	x_{42}
$p^* = 5.63$	0.778	1.31	1.67	1.22	0	1.22	1.67	0
$p^* = 4.40$	1.27	1.35	2.41	1.28	0	1.28	2.41	0

$$x_{i2}^* = \begin{cases} x_{ic} - \frac{p}{2w_{i2}}, & \text{if } w_{i2} \geq \frac{p}{2x_{ic}}, \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

and

$$p^* = \begin{cases} \frac{X_c - N_a}{\tilde{w}_2}, & \text{or} \\ \frac{X_c - X_t - N_a + \sqrt{(X_c - X_t - N_a)^2 + 2w_1\tilde{w}_2}}{\tilde{w}_2}, \end{cases} \quad (7)$$

where $X_c = \sum_{i \in \mathcal{N}_2} x_{ic}$, $N_a = \sum_{i \in \mathcal{N}_1} 1$, $w_1 = \sum_{i \in \mathcal{N}_1} w_{i1}$, and $\tilde{w}_2 = \sum_{i \in \mathcal{N}_2} \frac{1}{w_{i2}}$. The sets are $\mathcal{N}_1 = \{i \in \mathcal{N} : w_{i1} \geq p\}$ and $\mathcal{N}_2 = \{i \in \mathcal{N} : w_{i2} \geq \frac{p}{2x_{ic}}\}$.

We obtain the optimization solution by using Lagrangian and KKT condition [5]. The detailed procedures are omitted.

IV. PERFORMANCE EVALUATION

There are one retailer and four customers in the grid. Their weights are $w_{11} = 10$, $w_{12} = 15$, $w_{21} = 15$, $w_{22} = 10$, $w_{31} = 2$, $w_{32} = 10$, $w_{41} = 15$, and $w_{42} = 1$. Also, $X_{\max} = 10$ kW and $x_{ic} = 1.5$ kWh for all i. The results are presented in Table I. When $p^* = 5.63$ȼ/kWh, all the consumed electricity is less than 10 while it is equal to 10 when $p^* = 4.40$ȼ/kWh. Customer 1 consumed more electricity to type 2 appliance than that of type 1, but customer 2 behaves the opposite due to their weight. Customers 3 and 4 consumed no electricity to type 1 and type 2 appliance, respectively, since their weight is very low. With proposed game structure, each customer wisely consumes its energy to maximize its payoff and the retailer can maximize its profit.

V. CONCLUSION

We proposed a demand response program based on the Stackelberg game between one retailer and many customers in a smart grid. With help of a two-way communication infrastructure, the retailer [a leader] broadcasts the electricity price information. Then, the customers [followers] consume electricity to maximize their benefit. We designed the payoff functions and showed the solution of the game, i.e., a Stackelberg equilibrium.

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