Stackelberg-Game-Based Demand Response for At-Home Electric Vehicle Charging

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***Abstract*—Consumer electricity consumption can be controlled through electricity prices, which is called demand response. Under demand response, retailers determine their electricity prices, and customers respond accordingly with their electricity consumption levels. In particular, the demands of customers who own electric vehicles (EVs) are elastic with respect to price. The interaction between retailers and customers can be seen as a game because both attempt to maximize their own payoffs. This study models an at-home EV charging scenario as a Stackelberg game and proves that this game reaches an equilibrium point at which the EV charging requirements are satisfied, and retailer profits are maximized when customers use our proposed utility function. The equilibrium of our game can vary according to the weighting factor for the utility function of each customer, resulting in various strategic choices. Our numerical results confirm that the equilibrium of the proposed game lies somewhere between the minimum-generation-cost solution and the result of the equal-charging scheme.**

***Index Terms*—Demand response, electric vehicle (EV), real-time pricing, Stackelberg game.**

I. Introduction

To DESIGN a more efficient and robust power grid, a new power-grid paradigm using information and communication technology, which is the so-called *smart grid*, has been considered [1]. Smart grid has the capability to adaptively control the electricity demand with the help of a real-time two-way communication system. Through the controllability of the electricity demand, electricity demand throughout the day can be characterized as a more stable form. In particular, it can shift the demand at peak hours to off-peak hours, facilitating measures to minimize the peak-to-average ratio (PAR) and generation costs.

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There are direct and indirect methods to control demand. As a direct control method, a retailer can control the electricity load based on its contract with customers [2]. This is an easier method to control the demand, but issues regarding privacy and flexibility do exist. In contrast, demand response,[1] such as applying different rates during different hours of the day, is an indirect method. In this method, customers can reasonably reduce their electricity charges by shifting their deferrable loads to lower price hours according to pricing strategies, such as critical-peak pricing, time-of-use pricing (TOU), and real-time pricing.

The importance of demand response increases when electric vehicles (EVs)[2] are prevalent because residences with EVs consume more electricity and react more elastically to price [3]. However, it is difficult to shape the deferrable loads into off-peak hours with current demand response programs. In addition, with a careless demand response design, responses from customers may synchronize, resulting in abrupt changes in aggregated loads at a certain low-price period [4].[3]

Another characteristic of demand response is that it may cause a conflict of interest between a retailer and its customers. A retailer selects its electricity price to maximize profit, and customers adjust their demands according to the announced price, the process of which can be regarded as a game [7]. Note that in reality, electricity retailers are significantly regulated by governments. However, we expect the electricity market to be deregulated to a certain degree. Regardless of whether such deregulation actually occurs, we cover the issue in Section V.

Several research works have been conducted in designing games for demand response [8]–[12]. In [8] and [9], energy consumption games are formulated to offer users an incentive to cooperate. The players in these games are users only. To converge to a Nash equilibrium, in [8], they exchange information about their electricity loads, and in [9], a utility company helps its users exchange their information. In addition, Stackelberg games between retailers and users were proposed in [10] and [11], where a retailer announces its electricity price to maximize its revenue, and each user consumes electricity accordingly to maximize their utility, respectively. However,

[1] In this paper, we use the term demand response as price-based demand response.

[2] In this paper, an EV indicates any battery-type EV, such as a hybrid EV (HEV), a battery EV, or a plug-in HEV.

[3] Despite this potential threat, both the grid and customers can benefit if EVs are viewed as distributed batteries. Although this is another active area in smart grid research [5], [6], it is beyond our scope. In this paper, we use EVs for charging only.

this is a one-shot game without the constraint of EV charging requirements. In [12], a game for an optimal TOU was designed without any user constraints. Gatsis *et al.* have also solved an energy scheduling problem for load control to minimize the total cost considering user constraints [13]. However, a retailer and its customers are assumed to cooperate, which is not actually a game scenario. To the best of our knowledge, none of the previous research works have dealt with EV charging as a sequential game.

Herein, we model the problem of demand response as a Stackelberg game between a retailer and its customers under the constraint of EV charging requirements. In our game, the retailer and its customers are the leader and followers, respectively. The retailer targets maximizing its profits by setting the proper electricity price while satisfying the charging requirements of each customer. Each customer consumes electricity according to the price announced by the retailer. To meet the charging requirements, we design a utility function that reflects the demand of each customer and prove the existence of equilibrium in our proposed game.

The proposed game provides various strategic choices depending on their weight to the utility function. The equilibria in our proposed game vary between an optimum policy solution and an equal-charging policy solution. The optimum policy aims at minimizing the costs only, whereas the equal-charging policy attempts to charge electricity at an equal rate throughout a given period. By assigning a different weight to a utility function, the proposed game can control the equilibrium. Numerical results confirm that the aggregated electricity consumption in our proposed game is maintained at a certain level between the results of the optimum policy and the equal-charging policy.

The remainder of this paper is organized as follows. We first describe our system model in Section II. Next, in Section III, the problem is modeled as a Stackelberg game, and the existence of equilibrium is proved. In Section IV, we find the optimum solution and compare it with our game result. After comparing our proposed scheme with other competitive schemes in Section V, we offer some concluding remarks in Section VI.

II. System Model

As shown in Fig. 1, we consider a set of customers, i.e., $\mathcal{N}$, that are served by a single retailer.[4] In this paper, the objective of a retailer is to maximize profit through the resale of electricity bought at a day-ahead market to its customers. We divide each day into H time periods and denote a period set as $\mathcal{H} = \{1, 2, \ldots, H\}$ and a period index as h. We assume that no congestion occurs in the distribution grids and that the retailer is connected to all generators and customers through both power and communication networks. In Fig. 1, the solid and dotted lines represent a power distribution network and a two-way communication network, respectively. Several communication candidates, such as WiMAX, WiFi, smart utility networks, and power-line communications, are available for the smart grid system [15].

[4]Note that, here, the term "retailer" is different from an "aggregator." A retailer aims to make a profit by selling electricity, whereas an aggregator is an agent for EV customers [14].

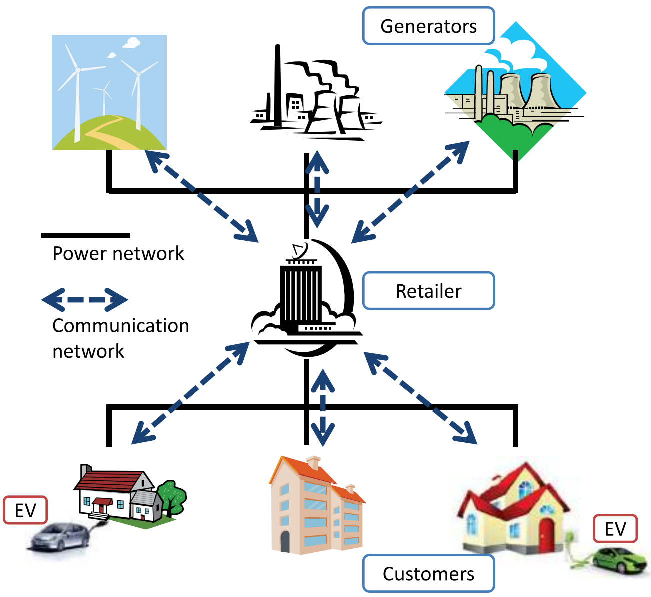

Fig. 1. Smart grid system model. Multiple generators and customers are connected through a single retailer. Some customers have EVs that are charged at home.

In this paper, we focus on the transactions between a retailer and its customers for charging EVs at home. The retailer makes a contract with its customers to supply electricity for EV charging. Therefore, the retailer should fulfill the EV charging requirements. At each h, the retailer announces the unit price of electricity p^h, and each customer i then responds with their demand x_i^h in kilowatts according to p^h. It is assumed that the EVs are charged only in residential areas. We designed a Stackelberg game between a retailer and its customers, where the leader (retailer) chooses the strategy prior to the decisions by the followers (customers) by knowing how the followers will react. With predictable electricity consumptions, the retailer can maximize its profit by shifting the EV charging loads from peak to off-peak hours.

A. Electricity Demand Model for Customers

Not all home appliances, such as lights and computers, can be controlled based on price. The devices do not respond to the price information, but other appliances such as EVs are considerably elastic to price, as long as their requirements are fulfilled.[5] For instance, a common requirement of a battery-type device is full charging of the battery before use. This can be modeled as

$$\begin{aligned} &\sum_{h \in \mathcal{T}_i} \mu_c x_i^h = E_i \\ &0 \leq x_i^h \leq \delta_i, \qquad \text{if } h \in \mathcal{T}_i \\ &x_i^h = 0, \qquad \text{if } h \notin \mathcal{T}_i \end{aligned} \tag{1}$$

[5]Appliances that need to meet customer satisfaction such as heat, ventilation, and air-conditioning (HVAC) systems are another type of price-elastic appliances. In this paper, we focus on EV-like elastic demand appliances and model other appliances simply as a certain base load.

where μ_c, E_i, and δ_i denote battery charging efficiency, the amount of electricity in kilowatt hours required for full charging of an EV,[6] and the maximum charging rate of customer i in kilowatts, respectively. The charging period starts at S_i and ends at F_i. Let $\mathcal{T}_i = [S_i, F_i]$ denote the set of time periods during which customer i charges a device.

Even if two charging appliances consume the same amount of electricity during a 1-h period, the degree of satisfaction may differ depending on how urgently each appliance needs to recharge. For example, take an appliance that has to charge 10 kWh within 10 h and another appliance that has to charge 2 kWh within 5 h. When 0.5 kWh is charged in 1 h, the former appliance needs more electricity per hour, whereas the latter is satisfied. To express the degree of satisfaction when an appliance charges x amount of electricity in an hour, we define a utility function as a function of x. Here, we assume that 1) the utility function is nondecreasing, 2) the marginal satisfaction decreases as x increases, 3) the utility function has a limit, and 4) a higher weight provides more satisfaction to the customer for the same amount of electricity consumption.

We use a utility function modified from that described in [17], which is a modified version of a quadratic function with linearly decreasing marginal satisfaction. We formally define the utility function of customer i as

$$U_i(x_i, w_i, \delta_i) = \begin{cases} w_i x_i - \frac{w_i}{2\delta_i} x_i^2, & \text{if } 0 \le x_i \le \delta_i \\ \frac{w_i \delta_i}{2}, & \text{if } x_i \ge \delta_i \end{cases} \quad (2)$$

where x_i and w_i are the electricity consumption in kilowatts and the weight of customer i, respectively. We assume that w_i does not change over time.

Note that unlike [17], the maximum satisfaction point does not change with the weight in our proposed utility function but depends only on δ_i. Our proposed utility function always has maximum utility at $x_i = \delta_i$. Intuitively, the customer's satisfaction does not continue to increase when the customer consumes more electricity than δ_i.

Fig. 2 shows three examples of a utility function. The parameter values are $w_1 = 10$, $w_2 = 20$, $w_3 = 20$, $\delta_1 = 1$, $\delta_2 = 1$, and $\delta_3 = 2$, where w_i is the weight for customer i. The utilities of customers 1 and 2 do not increase from 1 kWh, and the same level of electricity consumption yields different results because of different weights. Customers 2 and 3 have the same weight but have different utilities for the same electricity consumption since their δ_i values are different. Basically, higher w_i and δ_i values result in higher utility for the same x_i.

B. Electricity Cost Model for Retailer

When a retailer buys electricity from generators, the cost of electricity is different over time. In general, generating more electricity requires higher marginal cost. Since modeling the cost function according to the amount of electricity is still a research issue [18] but is beyond the scope of this paper, here, we assume that the cost function is an increasing, convex, and differentiable function with respect to electricity consumption.

[6]To use a battery more efficiently, the operation range of the battery is limited to about 65%, i.e., from 20% to 85%, of the battery capacity [16].

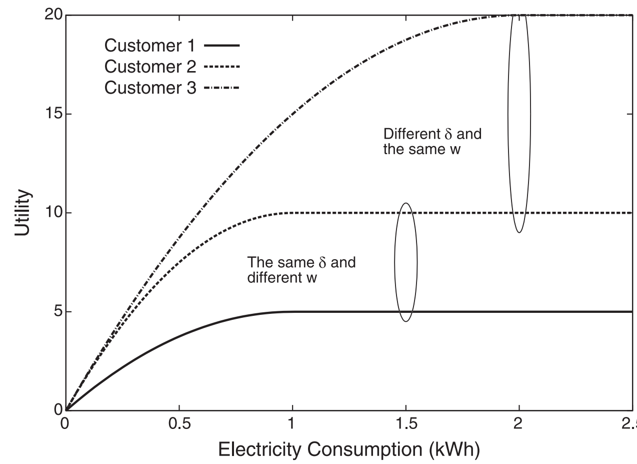

Fig. 2. Example utility functions. The parameters are set to $w_1 = 10$, $w_2 = 20$, $w_3 = 20$, $\delta_1 = 1$, $\delta_2 = 1$, and $\delta_3 = 2$.

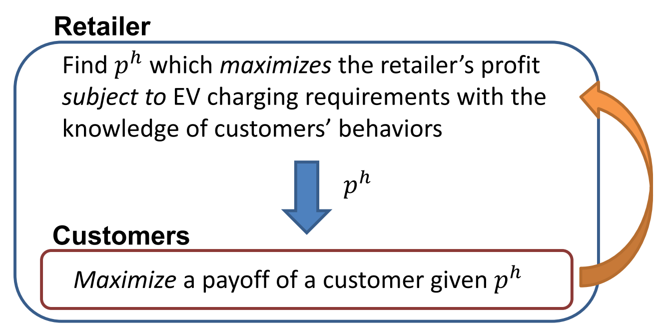

Fig. 3. Proposed Stackelberg game structure.

III. Stackelberg Game Design

We design a Stackelberg game with one retailer (leader) and many customers (followers). The overall problem structure is presented in Fig. 3. Initially, the retailer selects a set of prices to maximize its profit based on its knowledge of customer behaviors. Each customer then chooses their electricity consumption level according to the electricity price. We start by analyzing the customer behaviors and then do the same for the retailer, which is a backward induction technique for deriving an equilibrium point of the Stackelberg game. The optimal solution for each part therefore forms the equilibrium (i.e., subgame perfect equilibrium) of the game.

A. Analysis of the Customer Side

Each customer chooses their electricity consumption level to maximize their benefit, given electricity price p announced by the retailer. It is assumed that the weight for each customer is preassigned. Proposition 1 in this section covers how each customer chooses their weight. We define the payoff function $g_i(\cdot)$ of customer i as

$$g_i(x) = U_i(x_i, w_i, \delta_i) - p x_i. \quad (3)$$

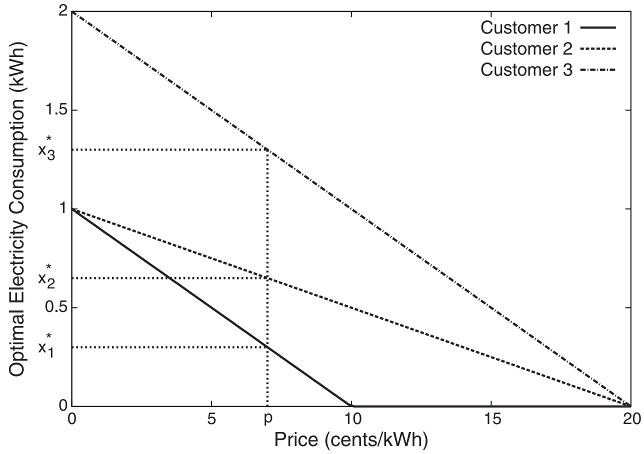

Fig. 4. Best response functions of customers. We set the parameters to $w_1 = 10$, $w_2 = 20$, $w_3 = 20$, $\delta_1 = 1$, $\delta_2 = 1$, and $\delta_3 = 2$.

The meaning of the payoff function is straightforward. The first term is a utility value when the customer consumes x_i amount of electricity with w_i, and the second term is the cost of the electricity consumption. As the utility function is differentiable and the second term is linear, the payoff function is also differentiable. We obtain the first-order derivative of $g_i(\cdot)$ as

$$\frac{dg_i}{dx_i} = \begin{cases} w_i - p - \frac{w_i}{\delta_i} x_i, & \text{if } 0 \le x_i \le \delta_i \\ -p, & \text{if } x_i \ge \delta_i. \end{cases} \quad (4)$$

Its maximum is achieved when $dg_i/dx_i = 0$. Therefore, we can define the best response function to the given p as

$$x_i^*(p) = \begin{cases} \delta_i \left(1 - \frac{p}{w_i}\right), & \text{if } p \le w_i \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Each customer i consumes the optimal amount of electricity x_i^* according to their weight, the maximum charging rate, and the announced electricity price. Fig. 4 shows the best response functions of the three customers in Fig. 2. For customers 1 and 3, their ratios of δ_i to w_i are the same, which creates the same slope. Customers 1 and 2 have the same y-intercept. For a given p, the optimal electricity consumption levels of customers 1, 2, and 3 are x_1^*, x_2^*, and x_3^*, respectively.

B. Analysis of the Retailer Side

The retailer determines electricity price p to maximize its profit, knowing that each customer will consume $x_i^*(p)$. It is assumed that the retailer knows the requirements of each customer, such as E_i, $\mathcal{T}_i$, δ_i, and w_i before setting the price.[7] Let $\mathbf{p}$ be a vector of prices announced by the retailer, that is, $\mathbf{p} := [p^1, p^2, \ldots, p^H]$. This is the only control variable of the retailer. We define the payoff function $f(\cdot)$ of the retailer as

$$f(p^h) = R(p^h) - C(X^h) \quad (6)$$

[7]Section III-C explains how the prediction errors affect our game's performance.

where $R(\cdot)$ and $C(\cdot)$ are the revenue and cost functions, respectively. The revenue function is simply defined as $R(p^h) = p^h \sum_{i\in\mathcal{N}} x_i^h$, that is, the price multiplied by the amount of electricity consumed by the customers. We use a quadratic cost function [19] as $C(X^h) = a(X^h)^2$, which is an increasing, concave, and differentiable function with respect to electricity consumption. X^h represents the total electricity consumption at h, which includes the electricity consumption by commercial and residential base loads and the price-elastic load of each customer. This can be expressed as $X^h = x_0^h + \sum_{i\in\mathcal{N}} x_i^h$, where x_0^h denotes the commercial and residential base loads at h.

We now formulate an optimization problem from the perspective of the retailer as follows:

$$\begin{aligned} (\mathbf{P}) \quad & \max_{\mathbf{p}} \quad \sum_{h\in\mathcal{T}} f(p^h) \\ & \text{subject to} \quad \sum_{h\in\mathcal{T}_i} \mu_c x_i^h = E_i \text{ for all } i \in \mathcal{N} \\ & \qquad\qquad p^h \ge 0 \text{ for all } h \in \mathcal{T} \end{aligned}$$

where $\mathcal{T} = \cup_{i\in\mathcal{N}} \mathcal{T}_i$. To guarantee a feasible solution, we consider the charging constraint of each customer i as $\delta_i|\mathcal{T}_i| \ge E_i/\mu_c$. This means that the amount of electricity when a customer charges at the maximum rate for the entire charging duration should be greater than or equal to the charging requirement. For the other customers, whose charging requirements are $\delta_i|\mathcal{T}_i| < E_i/\mu_c$, any charging scheme cannot satisfy the requirement. They are regarded as base loads since they are not price-elastic loads.

1) Single-Customer Case: We begin by solving **(P)** using a simple case, i.e., a single customer i with a base load. The problem can then be rewritten as

$$\begin{aligned} (\mathbf{P}1) \quad & \max_{\mathbf{p}} \quad \sum_{h\in\mathcal{T}_i} \left(p^h x_i^h - a\left(x_0^h + x_i^h\right)^2\right) \\ & \text{subject to} \quad \sum_{h\in\mathcal{T}_i} \mu c x_i^h = E_i \\ & \qquad\qquad p^h \ge 0 \text{ for all } h \in \mathcal{T}_i. \end{aligned}$$

Since $x^*(p)$ in (5) is always zero for $p > w_i$, the optimal solution cannot exist for $p > w_i$. Therefore, we limit the price domain to $0 \le p^h \le w_i$. Then, $x_i^h = \delta_i(1 - p^h/w_i)$, and we obtain the objective function as

$$\sum_{h\in\mathcal{T}_i} \left(-\left(\frac{\delta_i}{w_i} + a\frac{\delta_i^2}{w_i^2}\right)(p^h)^2 + \left(\delta_i + 2a\frac{\delta_i^2}{w_i}\left(1 + x_0^h\right)\right)p^h - a\delta_i^2\left(1 + x_0^h\right)^2 \right) \quad (7)$$

and the equality constraint as

$$\sum_{h\in\mathcal{T}_i} p^h = w\left(T_i - \frac{E_i}{\mu_c \delta_i}\right)$$

where $T_i = |\mathcal{T}_i|$. The objective function is a negative quadratic function that is concave, and all constraints are linear. Thus, the problem **(P1)** is a convex optimization problem. Because we assume that a feasible solution exists, there is a $\mathbf{p}$ such that $\sum_{h\in\mathcal{T}_i} p^h = w(T_i - (E_i/\mu_c\delta_i))$ and $0 \le p^h \le w_i$. This is Slater's condition, i.e., a sufficient condition for strong duality. Using a technique to solve the convex optimization problem, we obtain the solution $\mathbf{p}^*$.

The simplest case is that all the base loads are zeros, i.e., $x_0^h = 0$ for all $h \in \mathcal{T}_i$. The solution of this case is

$$p^{h*} = w_i\left(1 - \frac{E_i}{\mu_c\delta_i T_i}\right) \qquad \text{for all } h \in \mathcal{T}_i \tag{8}$$

and the actual electricity consumption of the customer becomes

$$x_{i,g}^{h*} = \frac{E_i}{\mu_c T_i} \qquad \text{for all } h \in \mathcal{T}_i \tag{9}$$

where subscript g indicates the "game." The detailed procedures are presented in Appendix A.

With nonzero base loads, its solution is heterogeneous. We obtain the solution $\mathbf{p}^*$ and the actual electricity consumption as

$$p^{h*} = \begin{cases} w_i, & \text{if } h \in \mathcal{T}_{\min}^g \\ \frac{w_i}{T_{\text{int}}^g}\left(T_{\max}^g + T_{\text{int}}^g - \frac{E_i}{\mu_c\delta_i}\right) & \\ +\frac{a}{1+a\delta_i/w_i}\left(x_0^h - \frac{1}{T_{\text{int}}^g}\sum\limits_{h\in\mathcal{T}_{\text{int}}^g} x_0^h\right), & \text{if } h \in \mathcal{T}_{\text{int}}^g \\ 0, & \text{if } h \in \mathcal{T}_{\max}^g \end{cases} \tag{10}$$

$$x_{i,g}^{h*} = \begin{cases} 0, & \text{if } h \in \mathcal{T}_{\min}^g \\ \frac{E_i/\mu_c - T_{\max}^g\delta_i}{T_{\text{int}}^g} & \\ -\frac{a\delta_i/w_i}{1+a\delta_i/w_i}\left(x_0^h - \frac{1}{T_{\text{int}}^g}\sum\limits_{h\in\mathcal{T}_{\text{int}}^g} x_0^h\right), & \text{if } h \in \mathcal{T}_{\text{int}}^g \\ \delta_i, & \text{if } h \in \mathcal{T}_{\max}^g \end{cases} \tag{11}$$

respectively, where $\mathcal{T}_{\min}^g$, $\mathcal{T}_{\text{int}}^g$, and $\mathcal{T}_{\max}^g$ are a partition[8] of $\mathcal{T}_i$, and their cardinal numbers are $T_{\min}^g$, T_{int}^g, and $T_{\max}^g$, respectively. We define the partition as follows:

$$\begin{aligned} \mathcal{T}_{\min}^g &= \left\{h \in \mathcal{T}_i : x_0^h > \frac{1}{T_{\text{int}}^g}(w_i + a\delta_i)\left(T_{\max}^g - T_i - \frac{E_i}{\mu_c\delta_i}\right)\right. \\ &\qquad \left. + \frac{1}{T_{\text{int}}^g}\sum_{h\in\mathcal{T}_{\text{int}}^g} x_0^h + \frac{w_i}{a} + \delta_i\right\} \\ \mathcal{T}_{\max}^g &= \left\{h \in \mathcal{T}_i : x_0^h < \frac{1}{T_{\text{int}}^g}(w_i + a\delta_i)\left(T_{\max}^g - T_i - \frac{E_i}{\mu_c\delta_i}\right)\right. \\ &\qquad \left. + \frac{1}{T_{\text{int}}^g}\sum_{h\in\mathcal{T}_{\text{int}}^g} x_0^h\right\} \\ \mathcal{T}_{\text{int}}^g &= \mathcal{T}_i \setminus (\mathcal{T}_{\min}^g \cup \mathcal{T}_{\max}^g). \end{aligned}$$

[8] $\mathcal{T}_i = \mathcal{T}_{\min}^g \cup \mathcal{T}_{\text{int}}^g \cup \mathcal{T}_{\max}^g$, and there is no intersection between any two distinct sets in $\{\mathcal{T}_{\min}^g, \mathcal{T}_{\text{int}}^g, \mathcal{T}_{\max}^g\}$.

Note that when a base load at h is higher than a certain threshold, additional electricity consumption is very critical. Thus, the retailer sets a very high price to prevent further electricity consumption. This period is $\mathcal{T}_{\min}^g$. On the other hand, when a base load at h is lower than another threshold, having the customer consume as much electricity as possible is a good method to maximize the retailer's payoff. This period is $\mathcal{T}_{\max}^g$. In reality, $\mathcal{T}_{\min}^g$ is virtually zero.

We compare the game results with the optimum solution in Section IV.

2) N-Customer Case: Using the previous result, we extend our formulation to cover a general case of N customers with a base load. This is a case of problem **(P)**. Although problem **(P)** is not convex, we can treat it as a convex optimization problem if the following constraint is added:

$$p^h \le \min\{w_n : \text{for all } n \in \mathcal{N}\}, \text{ for all } h \in \mathcal{T}. \tag{12}$$

With this constraint, $R(p^h) = \sum_{h\in\mathcal{T}} p^h \sum_{n\in\mathcal{N}} x_n^h$ becomes a concave function since $x_n^h = \delta_n(1 - p^h/w_n)$ for all $n \in \mathcal{N}$. In addition, since the composition of two concave functions is concave, $C(X^h) = C(x_0^h + \sum_{n\in\mathcal{N}} x_n^h)$ is a concave function. Therefore, the problem becomes a convex optimization problem.

However, we need to consider one more aspect to obtain the solution to **(P)**. Since the charging requirement constraint of each customer is a strict equality condition and a single price p^h is announced to all customers, **(P)** is not guaranteed to have a solution. To guarantee a solution, we design a w-generation function $h(\cdot)$.

Proposition 1: If each customer i uses the following w-generation function:

$$w_i = h(E_i, \delta_i, T_i) = w_{\mathbf{ref}}\frac{\alpha}{1 - E_i/\mu_c\delta_i T_i} \tag{13}$$

where $w_{\mathbf{ref}}$ and α are constant parameters, and problem **(P)** with constraint (12) always has a solution.

The proof is given in Appendix B. By using the w-generation function, customers are motivated to join the game, because they can decrease the electricity cost compared with other customers not participating in the game, which are regarded merely as base load in the game. This benefit will be numerically shown in Section V-B.

Note that the parameters $w_{\mathbf{ref}}$ and α represent the customer's willingness to charge their battery and a normalization parameter for the denominator, respectively.

A closed-form solution to problem **(P)** is unique but expressed using customer indexes according to the observed order of customers. We derive the solution for two customers out of N in Appendix B. This is still a convex optimization problem; hence, we can obtain its solution using the CVX package[9] [20].

C. Prediction Uncertainty

Thus far, our proposed game is analyzed with static variables, which are precisely predicted using the day-matching method,

[9] CVX is a MATLAB-based modeling system for convex optimization.

the regression method [21], and the artificial neural network [22]. To deal with prediction uncertainty, we investigate its effect on our game.

To solve our proposed game, the required information is E_i, $\mathcal{T}_i$, δ_i, and w_i for $\forall\, i \in \mathcal{N}$ and base load x_0^h for $\forall\, h \in \mathcal{H}$. Among these, δ_i and w_i are given to the retailer since δ_i is static, and customers participating in the game will use the w-generation function. Therefore, we need to predict E_i, $\mathcal{T}_i$, and x_0^h. To capture the possible error, we model the prediction error for E_i as a Gaussian random variable ϵ_i with zero mean and variance σ_i^2. Then, the charging requirement of customer i is expressed as

$$E_{iR} = E_i + \epsilon_i \quad \forall\, i \in \mathcal{N} \tag{14}$$

where E_{iR} and E_i denote the actual charging requirement and the predicted requirement for customer i, respectively.

Similarly, we model the prediction error for x_0^h as a Gaussian random variable ϵ_0 with zero mean and variance σ_0^2. Then, we have

$$x_{0R}^h = x_0^h + \epsilon_0^h \quad \forall\, h \in \mathcal{H} \tag{15}$$

where x_{0R}^h and x_0^h denote the actual base load and the predicted base load at h, respectively.

For the charging period $\mathcal{T}_i$, since we model it as an integer variable, the Gaussian random prediction model cannot be used. Recent research has shown that its average prediction error is about 3%, and the ratio for more than 10% prediction error is less than 5% [23]. Therefore, to compensate the prediction error for $\mathcal{T}_i$, we simply use a reduced charging period by 1 or 2 h. The results with such prediction errors are shown in Section V-E.

IV. Comparison With Optimum and Game Solutions

Here, we formulate and solve the optimum problem and compare its solution with our proposed game result.

A. Optimum Solution

To evaluate the efficiency of the equilibrium of our proposed Stackelberg game, the optimum policy is considered as a reference. In our game, both the retailer and customers have different interests, resulting in equilibrium through price signaling. The optimum policy, however, simply aims at minimizing the generation cost while satisfying the charging requirements of the customers. Therefore, price signaling is not needed to control their electricity consumption. It is assumed that their electricity consumption is directly controlled.

We define the new problem as

$$\begin{aligned}
(\mathbf{S}) \quad & \min_{\mathbf{x}} \quad \sum_{h \in \mathcal{T}} C(X^h) \\
\text{subject to} \quad & \sum_{h \in \mathcal{T}_n} x_n^h = E_n, \ \text{for all } n \in \mathcal{N} \\
& 0 \le x_n^h \le \delta_n, \ \text{for all } h \in \mathcal{T}_n \text{ and } n \in \mathcal{N}.
\end{aligned}$$

This is a convex optimization problem. Note that, in real-life operations, the optimum solution is not achievable because all information on the base loads should be given in advance.

1) Single-Customer Case: The objective function becomes $a(x_0^h + x_i^h)^2$. We obtain the solution as

$$x_{i,o}^{h*} = \begin{cases} 0, & \text{if } h \in \mathcal{T}_{\min}^o \\ \frac{E_i/\mu_c - T_{\max}^o \delta_i}{T_{\text{int}}^o} \\ \quad - \left(x_0^h - \frac{1}{T_{\text{int}}^o} \sum\limits_{h \in \mathcal{T}_{\text{int}}} x_0^h \right), & \text{if } h \in \mathcal{T}_{\text{int}}^o \\ \delta_i, & \text{if } h \in \mathcal{T}_{\max}^o \end{cases} \tag{16}$$

where $\mathcal{T}_{\min}^o$, $\mathcal{T}_{\text{int}}^o$, and $\mathcal{T}_{\max}^o$ are a partition of $\mathcal{T}_i$, and their cardinal numbers are $T_{\min}^o$, T_{int}^o, and $T_{\max}^o$, respectively. Subscript o represents an "optimum" value. The sets are given as follows:

$$\begin{aligned}
\mathcal{T}_{\min}^o &= \left\{ h \in \mathcal{T}_i : x_0^h > \frac{1}{T_{\text{int}}^o} \sum_{h \in \mathcal{T}_{\text{int}}^o} x_0^h + \frac{E_i/\mu_c - T_{\max}^o \delta_i}{T_{\text{int}}^o} \right\} \\
\mathcal{T}_{\max}^o &= \left\{ h \in \mathcal{T}_i : x_0^h > \frac{1}{T_{\text{int}}^o} \sum_{h \in \mathcal{T}_{\text{int}}^o} x_0^h + \frac{E_i/\mu_c - T_{\max}^o \delta_i}{T_{\text{int}}^o} - \delta_i \right\} \\
\mathcal{T}_{\text{int}}^o &= \mathcal{T}_i \setminus \left(\mathcal{T}_{\min}^o \cup \mathcal{T}_{\max}^o \right).
\end{aligned}$$

Note that if all the base loads are zeros, i.e., $x_0^h = 0$ for all $h \in \mathcal{T}_i$, its solution is simply

$$x_{i,o}^{h*} = \frac{E_i}{\mu_c T_i}, \qquad \text{for all } h \in \mathcal{T}_i. \tag{17}$$

This is the same as the equilibrium of the game in (9).

Since this optimization problem only aims to minimize the generation cost, it attempts to make the hourly electricity consumption level as flat as possible. Therefore, if the base load is the same throughout the day, its solution is the same as in (17). For different hourly base loads, the results are also different. When the base load is low or high, customers consume more or less electricity, respectively.

However, there are upper and lower bounds of x_i^h. If the base load is lower than a threshold, consuming at the maximum rate, i.e., δ_i, is the optimum solution. In contrast, if the base load is higher than the other threshold, the customer should not consume any additional electricity. We define these thresholds as $\mathcal{T}_{\max}^o$ and $\mathcal{T}_{\min}^o$, respectively. The sums of $x_{i,o}^{h*}$ in $\mathcal{T}_{\max}^o$ and $\mathcal{T}_{\min}^o$ are $T_{\max}^o \delta_i$ and 0, respectively.

If the base load lies between the two thresholds, the consumption rate of customer i also lies somewhere between 0 and δ_i according to the base load level, and the total consumption is $\sum_{h \in \mathcal{T}_{\text{int}}^o} x_{i,o}^{h*} = E_i/\mu_c - T_{\max}^o \delta_i$. The first term of $x_{i,o}^{h*}$ in $h \in \mathcal{T}_{\text{int}}^o$ is $(E_i/\mu_c - T_{\max}^o \delta_i)/T_{\text{int}}^o$, which indicates the average electricity to be consumed in $\mathcal{T}_{\text{int}}^o$. The second term of $x_{i,o}^{h*}$ in $h \in \mathcal{T}_{\text{int}}^o$ is $x_0^h - (1/T_{\text{int}}^o) \sum_{h \in \mathcal{T}_{\text{int}}^o} x_0^h$, which indicates an offset induced by the base load at h. Since the second term is an offset, the sum of these second terms is zero. The second term is used to flatten the electricity consumption levels for $\mathcal{T}_i$. That is, if the base load at h is greater or less than average, the electricity consumption decreases or increases by the second term, respectively. Note that the base load plus the customer's electricity consumption in $\mathcal{T}_{\text{int}}^o$ is constant, i.e., $(E_i/\mu_c - T_{\max}^o \delta_i)/T_{\text{int}}^o + (1/T_{\text{int}}^o) \sum_{h \in \mathcal{T}_{\text{int}}^o} x_0^h$.

2) N-Customer Case: The objective function is $a(x_0^h + \sum_{n\in\mathcal{N}} x_i^h)^2$. Unlike the Stackelberg game, the minimization problem $(\mathbf{S})$ of the generation cost is a convex optimization problem. The solution is similar to that of the previous problem. Again, we can use CVX to obtain a solution to this problem.

B. Comparison

We now compare the result of our proposed game with the optimum solution. If there is no base load, the values are the same. Even with a certain base load, they are the same for $\mathcal{T}_{\min}$ and $\mathcal{T}_{\max}$. The result of the game and the optimum solution in $\mathcal{T}_{\text{int}}$ are different and are

$$x_{i,g}^{h*} = \frac{E_i/\mu_c - T_{\max}^g \delta_i}{T_{\text{int}}^g} - \frac{a\delta_i/w_i}{1 + a\delta_i/w_i}\left(x_0^h - \frac{1}{T_{\text{int}}^g}\sum_{h\in\mathcal{T}_{\text{int}}^g} x_0^h\right)$$

$$x_{i,o}^{h*} = \frac{E_i/\mu_c - T_{\max}^o \delta_i}{T_{\text{int}}^o} - \left(x_0^h - \frac{1}{T_{\text{int}}^o}\sum_{h\in\mathcal{T}_{\text{int}}^o} x_0^h\right)$$

respectively. The first terms, i.e., the average electricity consumption levels, of the two solutions are fixed. The second terms vary according to the base load level. The optimum solution entirely reflects the change of the base load, whereas the equilibrium of the game has a coefficient term of $(a\delta_i/w_i)/(1 + a\delta_i/w_i)$. Since a, δ_i, and w_i are positive, the coefficient is also positive and lies between 0 and 1. Although a and δ_i are given uncontrollable parameters, w_i is a controllable variable. When w_i decreases, the coefficient approaches 1, and thus, the equilibrium of the game approaches the optimum solution. On the other hand, when w_i increases, the coefficient approaches 0, and thus, the equilibrium of the game approaches the result of the case with no base load. This means that with a large w, customers are willing to pay more for electricity. The retailer, therefore, focuses on increasing the revenue without considering the generation cost.

V. Performance Evaluation

Here, we present numerical results for three scenarios: a single-customer scenario with an artificially injected base load and two large-scale scenarios with realistic simulation parameters. We compare our proposed Stackelberg game with the optimum policy and the simple policies of equal-rate charging and maximum-rate charging in terms of the generation cost and PAR.

A. Single-Customer Scenario

To compare the performance of our proposed game with that of the optimum policy in detail, we start with a simple scenario of a single customer with the parameters listed in Table I. We assume level-1 charging at home, which uses 120 V and 15 A, resulting in a 1.4-kW charging rate. The cost function parameters are set to $a = 0.2$.

Fig. 5 shows the electricity consumption for the single-customer scenario. The equilibria of our proposed games and the optimum solution vary according to the base load, which is represented by the solid line. The optimal solution shows a tendency of flattening the electricity consumption as much as possible. In this example, $\mathcal{T}_{\min}^o = \varnothing$, $\mathcal{T}_{\max}^o = [4, 8]$, and $\mathcal{T}_{\text{int}}^o = [1, 3] \cup [9, 10]$. The equilibria of the games show different tendencies as the weight varies. For a small weight of $w = 0.1$, the result is very close to that of the optimal solution. Since the revenue obtained from selling the electricity is small, the retailer's strategy is to minimize the generation cost. As w increases, the solution flattens because the revenue from selling the electricity becomes dominant in comparison to the generation cost. Thus, the retailer's strategy targets a maximization of revenue.

TABLE I
Parameter Values for Evaluation

	1-customer	420-same	420-different
E_i/μ_c (kWh)	10	11	7, 8, 9, 10, 11
δ_i (kW)	1.4	1.4	1.4, 1.5
S_i	1	7 PM	5, 6, 7, 8, 9 PM
F_i	10	6 AM	5, 6, 7 AM

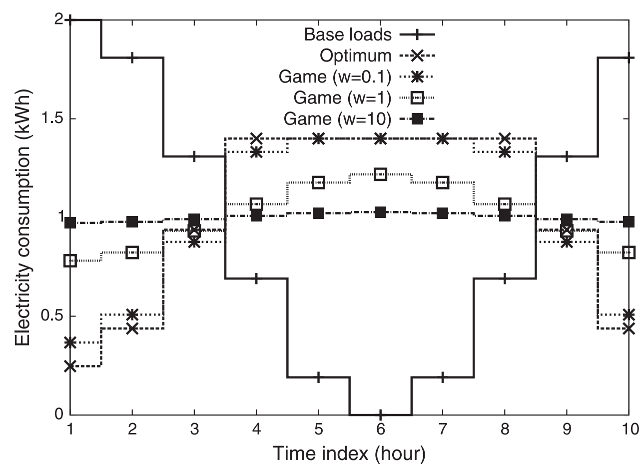

Fig. 5. Electricity consumption levels as a function of the base load. The solid line indicates the base load. The results of our proposed game and the optimum solution also vary according to the base load.

B. Large-Scale Scenario With Constant Parameters

For the large-scale scenario, the proposed game is evaluated through a distribution network from the IEEE 123-node test feeder, in which the total active power load is about 3500 kW [24]. It is assumed that 420 residences and other public or commercial buildings receive electricity from the feeder. The maximum loads of a single customer and commercial loads are assumed to be 6 and 850 kW, respectively [25]. Therefore, the maximum electricity consumption in the distribution network is about 3350 kW. The time period is divided into 24, i.e., $H = 24$. We assume that 80% of the residences have EVs and that all EVs are parked in their garages (start charging) after 7 P.M. and leave (end charging) before 6 A.M. Since our proposed game only considers the EV charging scenario at home of the customers, our time period of interest is from 7 P.M. to 6 A.M. Note that the following section describes the results of another simulation using a randomized parameter.

TABLE II
BASE LOADS IN UNITS OF KILOWATT HOURS

	Customer's base load			Commercial load
	Max	Min	HVAC	
5 PM	2.52	1.35	1.10	980
6 PM	3	1.68	1.10	980
7 PM	3	1.68	1.10	825
8 PM	3	1.68	1.10	665
9 PM	2.52	1.35	1.10	490
10 PM	1.68	0.66	1.10	410
11 PM	1.2	0.3	0.8145	410
0 AM	0.54	0.18	0.585	410
1 AM	0.3	0.12	0.307	410
2 AM	0.15	0.09	0	410
3 AM	0.15	0.09	0	410
4 AM	0.15	0.09	0	410
5 AM	0.3	0.12	0	410
6 AM	0.66	0.18	0	410
7 AM	1.01	0.49	0	410

At-home electrical devices can be classified into three types: those with inelastic base loads, HVAC base loads, and EV-like deferrable elastic loads. The first two types of loads consume their electricity probabilistically. The maximum and minimum levels of electricity consumption of inelastic base loads are listed in Table II. We randomly select the consumption level of the inelastic base load of each residence. Each customer chooses to turn on their HVAC system with a probability of 80%, the electricity consumption of which is provided in Table II. A commercial load, which is not controllable, is also given in Table II.

We set the cost parameter as $a = 7 \cdot 10^{-4}$. When the feeder supplies the maximum amount of electricity, the marginal electricity cost is $dC/dx|_{x=3500} = 4.9$ ¢/kWh $= 49$ \$/MWh. The real-time market clearing prices taken from the Midwest Independent Transmission System Operator range between 1.79 and 78.85 \$/MWh [26]. Since our cost parameter fits this range, it is a reasonable choice.

We evaluated a scenario in which all customers have the same EV parameters, which are presented under the category "420-same" in Table I; in addition, β in the w-generation function comes from the same EV parameters. Fig. 6 shows the aggregated electricity consumption level for this scenario. "Equal charging" and "ASAP charging" indicate that each EV is charged at an equal rate for the entire charging duration and with the maximum rate to finish charging as quickly as possible, respectively. In addition, "Optimum" and "ASAP charging" have the flattest and curviest lines, respectively. As shown in the single-customer scenario, our proposed games with $w = 0.1$ and 10 show almost the same results as "Optimum" and "Equal charging," respectively.[10] In the "ASAP charging" policy, all EVs finish their charging at 2 A.M., and thus, the aggregated electricity consumption level is very low starting from 3 A.M., resulting in a higher PAR.

Table III lists the generation cost, profit, and PAR. Since the marginal cost for the generation of electricity linearly increases with the electricity amount, the generation costs of "Optimum" and "ASAP charging" are the lowest and highest, respectively,

[10]Weight w here indicates the reference value.

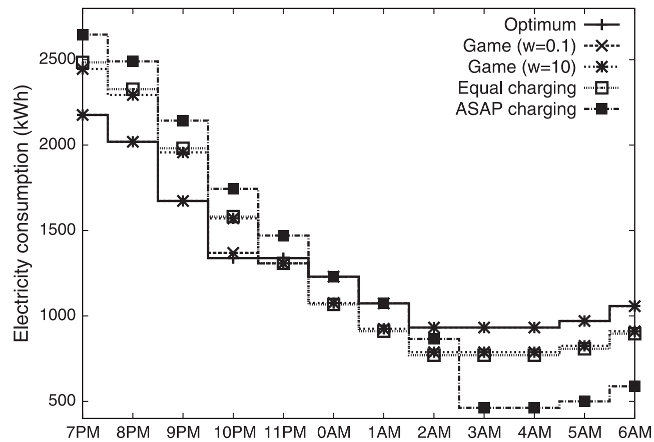

Fig. 6. Electricity consumption levels with the same EV charging requirement. All loads on the system are included. "ASAP charging" indicates charging a battery as soon as possible. "Optimum" and "ASAP charging" show the flattest and curviest lines, respectively.

TABLE III
RESULTS WITH THE SAME EV CHARGING REQUIREMENT

	Generation Cost (\$)	Profit (\$)	PAR
Optimum	232.3		1.675
Game ($w = 0.1$)	232.3	0.15	1.675
Game ($w = 10$)	247.0	126.3	1.755
Equal charging	249.1		1.783
ASAP charging	266.6		1.900

and PAR also shows the same tendency. Here, "Profit" is the same as (6). Since we do not know how high the set price will be, profits for "Optimum," "Equal charging," and "ASAP charging" were left blank. Compared with "ASAP charging," the "Optimum" policy and "Game $(w = 0.1)$" reduce the generation cost by about 14.8%. In addition, "Game $(w = 10)$" reduces the cost by approximately 7.9%. Similarly, the results of "Optimum," "Game $(w = 0.1)$," and "Game $(w = 0.1)$" show PAR improvements of 13.4%, 13.4%, and 7.9%, respectively. Weight w represents how much an EV customer is willing to pay for their electricity. Thus, the profit of the retailer increases as w increases, and the game result for a small w approaches the optimum solution. Note that the profit here only comes from the revenue generated by selling electricity for EV charging, and other profits from the base loads of the residences and commercial buildings are not considered in Table III.

To confirm the benefit of participating in the proposed game, we numerically obtained the increased cost when the EV owners are assumed to pay. Each customer can reduce its cost from 0.23 \$/EV to 0.13 \$/EV by participating in the game with $w = 0.1$. Hence, both the retailer and customers benefit from our proposed game compared with noncoordinated charging schemes.

C. Large-Scale Scenarios With Randomized Parameters

We evaluated a scenario wherein each parameter is randomly chosen from multiple options. The EV parameters are presented in Table I under the category of "420-different" [16], [27]. Major EV battery parameters come from Chevy Volt [16]. With 16-kWh capacity of the battery, 10.4 kWh is usable because the operational range of the battery is from 20% to 84.71%.

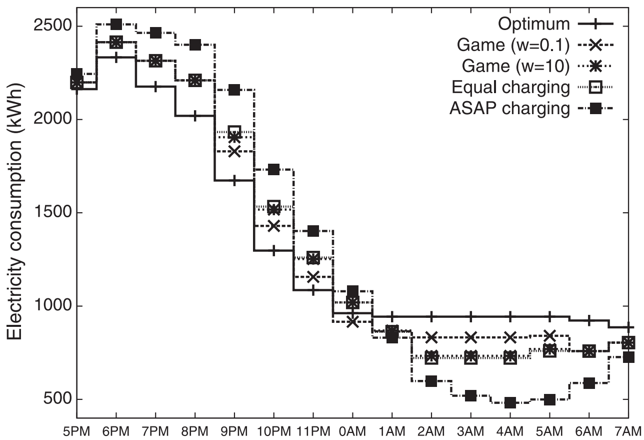

Fig. 7. Electricity consumption levels with various EV charging requirements. The general trend is the same as in the first scenario.

TABLE IV
PERFORMANCE COMPARISON FOR DIFFERENT EV CHARGING REQUIREMENTS

	Generation Cost ($)	Profit ($)	PAR
Optimum	221.3		1.729
Game ($w = 0.1$)	232.6	1.19	1.790
Game ($w = 10$)	235.4	147.0	1.790
Equal charging	236.1		1.790
ASAP charging	256.5		1.860

The battery charging efficiency is set to $\mu_c = 0.85$. Therefore, to fully charge the battery from the minimum state-of-charge, $10.4/0.85 = 12.2$ kWh is required. According to [27], the average daily vehicle mile of travel in 2009 is 28.97, which requires 9.41 kWh.[11] Then, we assume that E_i/μ_c is uniformly distributed in [7] and [11]. The arrival time range of [5 P.M., 9 P.M.] and the departure time range of [5 A.M., 7 A.M.] cover more than 90% of drivers [27]. We choose the constant parameters as $w_{\mathbf{ref}} = 0.1$ or 10 and $\alpha = 1$.

Fig. 7 shows the electricity consumption level. The general tendency is the same as for "420-same." Again, "Optimum" and "ASAP charging" show the flattest and curviest lines, respectively, and our proposed game results lie between them. In our proposed game with $w = 0.1$, the result shows similar behavior to the optimal solution.

Table IV lists the generation cost, profit, and PAR for this scenario. Compared with "ASAP charging," the "Optimum," "Game $(w = 0.1)$," and "Game $(w = 10)$" reduce the generation cost by 15.9%, 10.2%, and 8.9%, respectively. The improvement in PAR is also similar to the generation cost.

Fig. 8 shows the generation cost according to the penetration ratio. As the penetration ratio increases, the amount of electricity consumption increases, resulting in higher generation cost. The order for the five schemes, however, does not change with the penetration ratio. Note that the PAR result according to the penetration ratio shows similar tendency.

We also evaluated the performance of our proposed Stackelberg game with another large-scale scenario, which comes from an inland hot area [28]. Residences in the area consumes much

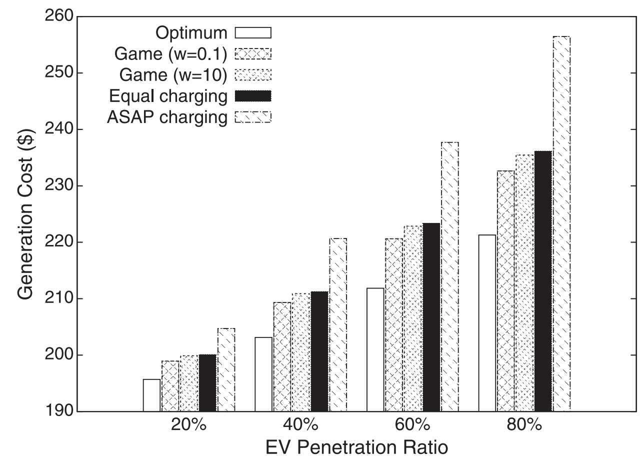

Fig. 8. Generation cost with various EV penetration ratios. Regardless of the penetration ratio, the order of generation cost does not change.

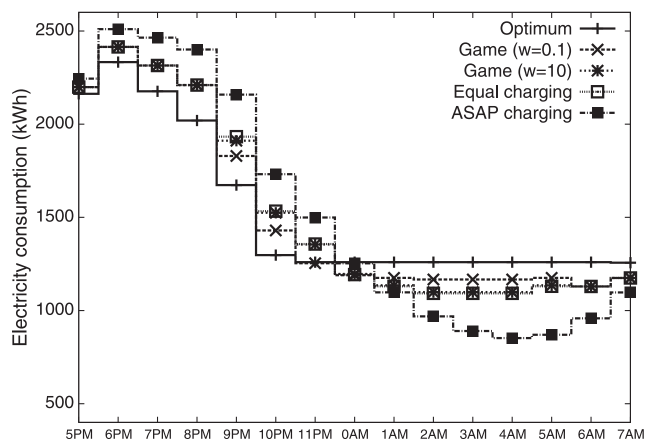

Fig. 9. Electricity consumption levels in an inland hot area. The general trend is the same as in the first scenario.

TABLE V
PERFORMANCE COMPARISON FOR AN INLAND HOT AREA

	Generation Cost ($)	Profit ($)	PAR
Optimum	263.9		1.522
Game ($w = 0.1$)	271.6	1.30	1.575
Game ($w = 10$)	273.1	147.5	1.575
Equal charging	273.4		1.575
ASAP charging	288.4		1.637

electricity from late evening to overnight. Thus, we raised the electricity consumption for HVAC to 1.1 kWh between 11 P.M. and 7 A.M.

Fig. 9 shows that the electricity consumption level and the general tendency of the other scenario is the same as the previous ones. Table V lists the generation cost, profit, and PAR for this scenario. Although the quantity of reduced generation cost becomes smaller, the "Optimum," "Game $(w = 0.1)$," and "Game $(w = 10)$" still outperform "ASAP charging." Because of increased electricity consumption during off-peak hours, PAR performance for all schemes is improved.

In conclusion, the optimum and ASAP charging policies always show the best and the worst performance, respectively. Our proposed Stackelberg game's performance lies between them, and its position can be adjustable with w.

[11]The average driving efficiency of Chevy Volt is 3.62 mi/kWh, and the battery charging efficiency is 0.85.

Because of the intrinsic nature of the electricity industry, electricity prices are generally regulated by governments [29]. Therefore, retailers are not allowed to set electricity prices too high. As we showed in our evaluation results, a game result with a small w approaches the optimum solution. On the other hand, a game with a large w results in a higher generation cost. Thus, through government regulations or retailer incentives, setting the upper bound of w properly leads to a desirable equilibrium.

D. Effect of w-Generation Function

Thus far, it is assumed that all EV customers follow our proposed w-generation function to fully charge their EV batteries. To verify the effect of the proposed w-generation function, we evaluated another case that some customers do not follow this function. We define another w-generation function, which is only decided by the amount of electricity to charge E_i as

$$w_{\mathbf{ref}} \frac{E_i}{C_{\text{norm}}} \tag{18}$$

where C_{norm} is a normalization constant. That is

$$C_{\text{norm}} = \frac{\sum_{i \in \mathcal{N}} w_{\mathbf{ref}} E_i}{\sum_{i \in \mathcal{N}} w_i} \tag{19}$$

where w_i comes from our proposed w-generation function.

Fig. 10 shows electricity consumption levels for two sample customers. Customer 1 does not finish its charging, and the amount of uncharged electricity is about 0.5 kWh. Although customer 2 fully charges its battery, the total generation cost for the electricity slightly increases compared with the previous result.

We simulated in case half of customers, i.e., 168, follow our proposed w-generation function and the other half customers follow the simple w-generation function. All the former half customers fully charge their EV while 123 customers out of the latter customers do. That is, 45 customers do not completely charge their EV batteries. The average and maximum uncharged capacities are 0.94 and 2.8 kWh, respectively, which correspond to 2.89 and 8.62 mi. Although the total amount of electricity consumption in this case is lower than that of the previous case, the total generation cost for this case is slightly higher than that of the previous case. This is because the latter half customers' consumed electricity is not properly distributed.

E. Effect of Prediction Uncertainty

We evaluated the above scenario with prediction errors. We set the standard deviation of ϵ_i as 3% since the prediction is quite accurate [23]. When $\epsilon_i < 0$, the EV of customer i fully charges its battery, and $|\epsilon_i|$ is not actually consumed. On the other hand, when $\epsilon_i > 0$, the battery of customer i is not fully charged by ϵ_i. However, the average uncharged capacities are 0.01 kWh, which is 0.0362 mi for the EV to drive. This is a negligible quantity. With this prediction error, the generation cost reduces 0.02% in "Game $(w = 0.1)$" and "Game $(w = 10)$."

We also simulated our proposed game with a reduced charging period. When the charging period reduces by 1 h, the generation costs for "Game $(w = 0.1)$" and "Game $(w = 10)$" change by 0.45% and 0.51%, respectively. When the charging period reduces by 2 h, they change by 0.82% and 0.72%, respectively. The reason why the generation cost hardly changes is that most of electricity is charged during off-peak hours, i.e., from 10 P.M. to 6 A.M., in our price-based game.

As the final prediction error, the prediction error for the base load x_0^h is considered. The prediction error ϵ_0 is added to x_0^h. If the standard deviation of ϵ_0 is set to 3%, the generation cost is almost the same as that without error. To confirm the adverse effect of the prediction uncertainty, the standard deviation of ϵ_0 is changed from 10% to 30%. Fig. 11 shows generation costs with prediction errors. As the prediction error increases, generation costs also increase without changing the order. When the prediction errors are severe (30%), the generation costs increase by about 7%. Conclusively, our proposed game works well, even with severe prediction uncertainties.

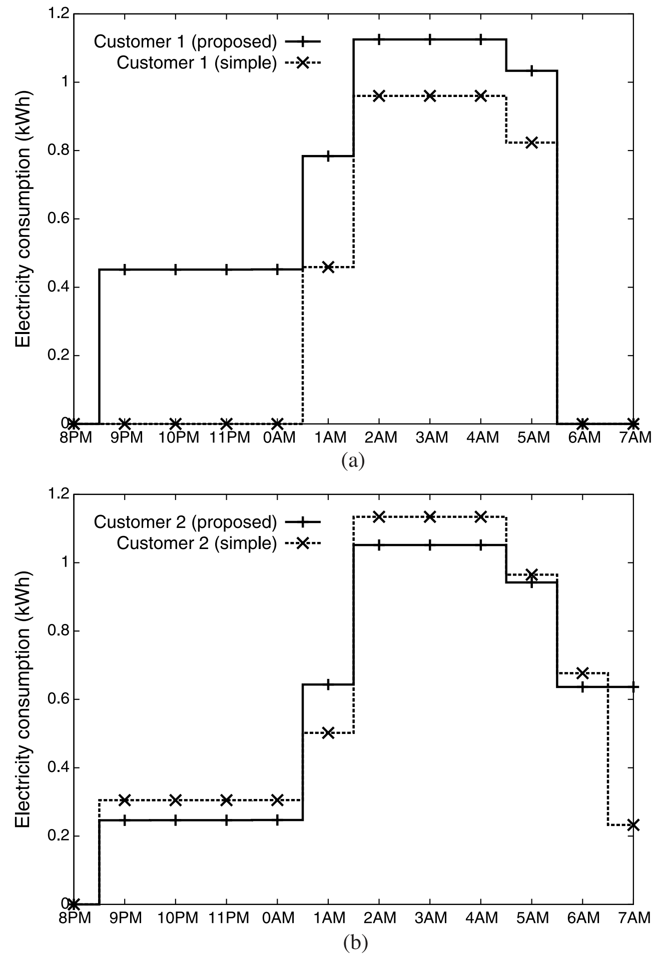

Fig. 10. Electricity consumption levels for two sample customers. With the simple w-generation function, customer 1 cannot fully charge its EV battery, while customer 2 can. However, the total generation cost slightly increases. (a) Customer 1. (b) Customer 2.

VI. CONCLUSION

As EVs become more popular, a large amount of deferrable load is being placed on power systems. Without a proper power system design, the peak demand will greatly increase,

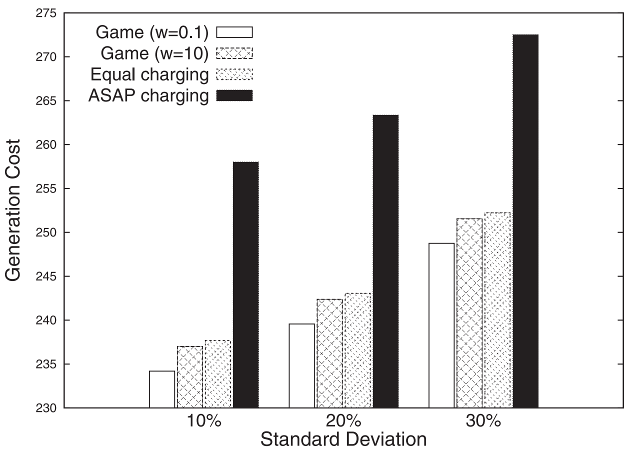

Fig. 11. Generation cost with prediction errors. Regardless of the prediction errors, our proposed game always outperforms the other schemes.

or another peak may occur. To control at-home EV charging, we formulated a demand response problem using a Stackelberg game that consists of a single retailer and its customers. In this game, both parties attempt to maximize their payoffs. Using the proposed utility function, we proved that the game always reaches an equilibrium point where the EV charging requirements are satisfied and the retailer's profits are maximized. According to the weight of the utility function of each customer, our game can achieve various results that lie between the optimum policy and equal-charging policy results.

Appendix A
Detailed Procedures to Solve (P1)

Here, we derive the solution to the convex optimization problem **(P1)**. The problem can be rewritten as a minimization problem by placing a negative sign on the objective function. Moreover, we change the charging requirement constraint with respect to p^h since $x_i^h = x_i^*(p^h) = \delta_i(1-(p^h/w_i))$

$$(\mathbf{P1}') \quad \min_{\mathbf{p}} \quad \sum_{h\in\mathcal{T}_i}\left(-p^h\delta_i\left(1-\frac{p^h}{w_i}\right)+a\left(\delta_i\left(1-\frac{p^h}{w_i}\right)\right)^2\right)$$

$$\text{subject to} \quad \sum_{h\in\mathcal{T}_i} p^h = w_i\left(T-\frac{E_i}{\mu_c\delta_i}\right)$$

$$0\le p^h \le w_i, \text{ for all } h\in\mathcal{T}_i.$$

The Lagrangian is

$$L(\mathbf{p},\lambda,\mu,\nu) = \sum_{h\in\mathcal{T}_i}\left(-p^h\delta_i\left(1-\frac{p^h}{w_i}\right)+a\left(\delta_i\left(1-\frac{p^h}{w_i}\right)\right)^2\right) + \sum_{h\in\mathcal{T}_i}\lambda^h\left(p^h-w_i\right) - \sum_{h\in\mathcal{T}_i}\mu^h p^h + \nu\left(\sum_{h\in\mathcal{T}_i}p^h - w_i\left(T-\frac{E_i}{\mu_c\delta_i}\right)\right)$$

where λ, μ, and ν are the Lagrangian multipliers. The optimal solution $\mathbf{p}^*$ must satisfy the Karush–Kuhn–Tucker (KKT) conditions [30], i.e.,

$$\sum_{h\in\mathcal{T}_i} p^{h*} = w_i\left(T_i-\frac{E_i}{\mu_c\delta_i}\right) \tag{20}$$

$$0\le p^{h*}\le w_i, \text{ for all } h\in\mathcal{T}_i \tag{21}$$

$$\lambda^{h*}\ge 0, \qquad \mu^{h*}\ge 0 \tag{22}$$

$$\lambda^{h*}\left(p^h-w_i\right)=0, \qquad \mu^{h*}p^h=0 \tag{23}$$

$$\frac{2\delta_i}{w_i}\left(1+a\frac{\delta_i}{w_i}\right)p^{h*}-\delta_i+2a\frac{\delta_i^2}{w_i}+\lambda^{h*}-\mu^{h*}+\nu=0. \tag{24}$$

To satisfy the slackness condition of (23), we can consider four possible cases:

i) $\lambda^{h*}=0$ and $\mu^{h*}=0$: Then, $0\le p^{h*}\le w_i$. Using two conditions in (20) and (24), we get

$$p^{h*}=w_i\left(1-\frac{E_i}{\mu_c\delta_i T_i}\right).$$

ii) $\lambda^{h*}=0$ and $\mu^{h*}>0$: Then, $p^{h*}=0$. This is the case of $T_i=(E_i/\mu_c\delta_i)$, so the customer always charges its EV with the maximum rate.

iii) $\lambda^{h*}>0$ and $\mu^{h*}=0$: Then, $p^{h*}=w_i$. The only case that satisfies this condition is $E_i=0$.

iv) $\lambda^{h*}>0$ and $\mu^{h*}>0$: This case has no p^{h*} that satisfies all the KKT conditions.

Appendix B
Proof of Proposition 1

Let us consider a case with no base load first. We will show that the solution of **(P1)** is the same as in the N-customer case. Suppose that there are N customers associated with a single retailer. Without loss of generality, we choose a customer i who has the minimum w_i among N customers. Then, the price solution of **(P1)** is

$$p^{h*}=w_i\left(1-\frac{E_i}{\mu_c\delta_i T_i}\right), \qquad \text{for all } h\in\mathcal{T}_i.$$

Because of the w-generation function, the ratio of w_i to w_j, for any other customer j, is

$$\frac{w_i}{w_j}=\frac{1-E_j/\delta_j T_j}{1-E_i/\mu_c\delta_i T_i}. \tag{25}$$

Using p^{h*} and (5), the electricity consumption of customer j is given as

$$x_{j,g}^{h*}=\delta_j\left(1-\frac{w_i}{w_j}\left(1-\frac{E_i}{\mu_c\delta_i T_i}\right)\right)=\frac{E_j}{T_j}.$$

Moreover, the total electricity consumption of customer j is

$$\sum_{h\in\mathcal{T}_j} x_{j,g}^{h*}=E_j$$

which satisfies the charging requirement. It means that all charging requirements of N customers are satisfied, so p^{h*} becomes the solution of **(P)** with no base load.

Now, let us consider the case with a base load. Similarly, assume that there are N customers associated with a single retailer. Without loss of generality, we choose customer i who has the minimum w_i among N customers. First, assuming that each $h \in \mathcal{T}_i$ is in $\mathcal{T}_{\rm int}^g$, we obtain the price solution $\mathbf{p}^*$ as

$$p^{h*} = \frac{w_i}{T_i}\left(T_i - \frac{E_i}{\mu_c \delta_i}\right) + \frac{a}{1 + a\delta_i/w_i}\left(x_0^h - \overline{x_0^h}\right)$$

where $\overline{x_0^h} = (1/T_i)\sum_{h\in\mathcal{T}_i} x_0^h$. For any other customer j, using p^{h*} and (5), we obtain the electricity consumption of customer j as

$$\begin{aligned} x_{j,g}^{h*}(p^{h*}) &= \delta_j\left(1 - \frac{w_i}{w_j}\left(1 - \frac{E_i}{\mu_c\delta_i T_i}\right) - \frac{a/w_j}{1+a\delta_i/w_i}\left(x_0^h - \overline{x_0^h}\right)\right) \\ &= \frac{E_j}{T_j} - \frac{a\delta_j/w_j}{1+a\delta_i/w_i}\left(x_0^h - \overline{x_0^h}\right). \end{aligned}$$

The total electricity consumption of customer j meets the charging requirement of

$$\sum_{h\in\mathcal{T}_j} x_{j,g}^{h*}(p^{h*}) = E_j.$$

Therefore, p^{h*} is the solution of **(P)** .

Next, we move on to the case that not all h are in $\mathcal{T}_{\rm int}^g$. The price solution $\mathbf{p}^*$ is

$$p^{h*} = \begin{cases} w_i, & \text{if } h \in \mathcal{T}_{\min}^g \\ \frac{w_i}{T_{\rm int}^g}\left(T_{\max}^g + T_{\rm int}^g - \frac{E_i}{\mu_c\mu_c\delta_i}\right) \\ +\frac{a}{1+a\delta_i/w_i}\left(x_0^h - \frac{1}{T_{\rm int}^g}\sum\limits_{h\in\mathcal{T}_{\rm int}^g} x_0^h\right), & \text{if } h \in \mathcal{T}_{\rm int}^g \\ 0, & \text{if } h \in \mathcal{T}_{\max}^g. \end{cases} \tag{26}$$

Using $\mathbf{p}^*$ and (5), the electricity consumption of customer j is given as

$$x_{j,g}^{h*} = \begin{cases} \delta_j\left(1 - \frac{w_i}{w_j}\right), & \text{if } h \in \mathcal{T}_{\min}^g \\ \delta_j\left(1 - \frac{w_i}{w_j T_{\rm int}}\left(T_{\max} + T_{\rm int} - \frac{E_i}{\mu_c\delta_i}\right)\right. \\ \quad \left. - \frac{a}{1+a\delta_i/w_i}\left(x_0^h - \overline{x_0^h}\right)\right), & \text{if } h \in \mathcal{T}_{\rm int}^g \\ \delta_j, & \text{if } h \in \mathcal{T}_{\max}^g. \end{cases} \tag{27}$$

Moreover, the total electricity consumption of customer j is

$$\begin{aligned} \sum_{h\in\mathcal{T}_i} x_{j,g}^{h*} &= \sum_{h\in\mathcal{T}_{\min}} x_{j,g}^{h*} + \sum_{h\in\mathcal{T}_{\rm int}} x_{j,g}^{h*} + \sum_{h\in\mathcal{T}_{\max}} x_{j,g}^{h*} \\ &= T_{\min}\delta_j\left(1 - \frac{w_i}{w_j}\right) + T_{\max}\delta_j \\ &\quad + \delta_j\left(T_{\rm int} - \frac{w_i}{w_j}\left(T_{\max} + T_{\rm int} - \frac{E_i}{\mu_c\delta_i}\right)\right) \\ &= \delta_j T_i - \frac{w_i}{w_j}\delta_j T_i\left(1 - \frac{E_i}{\mu_c\delta_i T_i}\right) \\ &= E_j\frac{T_i}{T_j}. \end{aligned}$$

In case of $\mathcal{T}_j = \mathcal{T}_i$, $\sum_{h\in(\mathcal{T}_j=\mathcal{T}_i)} x_{j,g}^{h*} = E_j$. In case of $\mathcal{T}_j \supset \mathcal{T}_i$, $\sum_{h\in\mathcal{T}_j} x_{j,g}^{h*} = \sum_{h\in\mathcal{T}_i} x_{j,g}^{h*} + \sum_{h\in(\mathcal{T}_j\setminus\mathcal{T}_i)} x_{j,g}^{h*}$. This is a new small problem, and it can be defined as

$$\begin{aligned} (\mathbf{SP}) \quad & \max_{\mathbf{p}} \quad \sum_{h\in\mathcal{T}_j\setminus\mathcal{T}_i}\left(p^h x_j^h - a\left(x_j^h\right)^2\right) \\ & \text{subject to} \quad \sum_{h\in\mathcal{T}_j\setminus\mathcal{T}_i} x_j^h = E_j\left(1 - \frac{T_i}{T_j}\right) \\ & \qquad 0 \le p^h \le w_j, \text{ for all } h \in \mathcal{T}_j \setminus \mathcal{T}_i. \end{aligned}$$

Thus, we can choose proper p^{h*} to satisfy the new charging requirement for $h \in \mathcal{T}_j \setminus \mathcal{T}_i$ with the same method. Therefore, p^{h*} becomes the solution of **(P)** with a base load. Note that the problem can be changed to cover the other case for all $h \in \mathcal{T}_{\rm int}^g$.

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