Topological and Statistical Analysis for the High-Voltage Transmission Networks in the Korean Power Grid

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ABSTRACT

A power grid is one of the most complex networks and is critical infrastructure for society. To understand the characteristics of a power grid, complex network analysis has been used from the early 2000s mainly for US and European power grids. However, since the power grids of different countries might have different structures, the Korean power grid needs to be examined through complex network analysis. This paper performs the analysis for the Korean power grid, especially for high-voltage transmission networks. In addition, statistical and small-world characteristics for the Korean power grid are analyzed. Generally, the Korean power grid has similar characteristics to other power grids, but some characteristics differ because the Korean power grid is concentrated in the capital area.

Key Words : Complex network analysis, Graph theory, Power grid

Ⅰ. Introduction

Since the end of the 20th century, complex network analysis has been attracting attention[1]. Traditionally, manufactured infrastructures have been component-wisely analyzed using physical law or mathematics, and remarkable progress has been made during the 20th century. Since today's society has become more connected, however, the component-wise analysis have limit for the intertwined issue. Whereas, for complex network analysis, intertwined issues are analyzed from a network viewpoint. For example, biological networks[2] and computer networks[3] have been analyzed using this approach with considerable progress. Also, graph theory is used in communication theory[23-25].

A power grid is one of the most complex networks and is critical infrastructure for society.

Therefore, complex network analysis is an efficient tool for power grid analysis. Researches on power grids using complex network analysis have been carried out since the early 2000s. For example, the actual power grids of the US[4], Italy[5], Europe[6], and India[7] have been analyzed for high-voltage transmission networks, while the medium and low voltage transmission network has been investigated for the Netherland power grid[8].

Recently, a helpful survey paper has been published on complex network analysis for a power grid[9]. The work has shown that some of the topological characteristics of the power grids of different countries are similar while others differ. The similar characteristics are due to the natural characteristics of the power system, and the different characteristics are due to the different geographical locations, development history, and morphological characteristics.
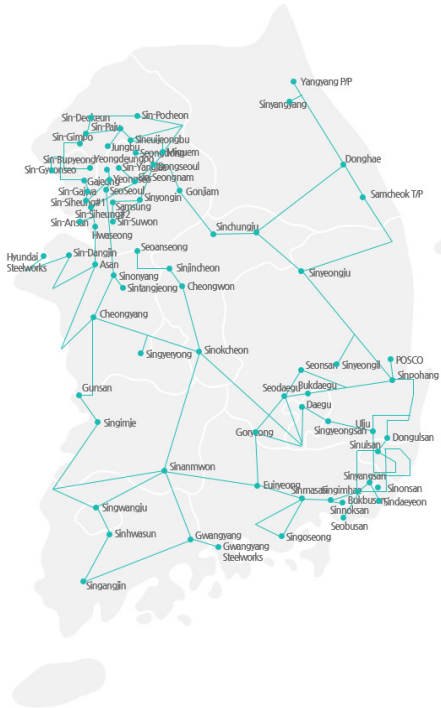
Fig. 1. Transmission map for 345 kV in Korea [11].

The Korean power grid for the high-voltage transmission consists of three voltage levels: 154 kV, 345 kV, and 765 kV. The first transmission line in Korea was established in 1935 using 154 kV, which had run between Seoul and Pyongyang. In 1976 and 2002, the operations of high-voltage transmission lines of 345 kV and 765 kV, respectively, were initiated[10]. Note that two high-voltage direct current (HVDC) transmission lines currently run between Jeju Island and Korean Peninsula, but these are not analyzed here because of their simple topology.

The topology of the 765 kV transmission network is a simple linear structure since its main role is delivering electricity to the capital area. On the other hand, the topologies of the 154 kV and 345 kV transmission networks are ring-shaped, especially in the capital area[10]. Fig. 1 shows the transmission map for 345 kV[11], showing that the transmission networks are concentrated in the capital and south-east parts of Korea.

In this paper, the Korean power grid is analyzed using a complex network approach. While this has

previously been investigated for the Korean power grid[12], because the work has focused on the US power grid, the Korean power grid has been only examined in terms of three topological characteristics with limited data. On the other hand, our work has three main analysis results for the 154 kV, 345 kV, and 765 kV transmission networks. We first analyze the topological characteristics of the Korean power grid such as average node degree, Pearson correlation coefficient, network efficiency, and clustering coefficient. Then, the statistical characteristics, i.e. distributions for nodal degree and shortest path, are analyzed. Finally, we examine the small-world network characteristic for the Korean power grid.

The remainder of this paper is organized as follows. We first briefly describe the topological characteristics in Section 2. We then analyze the Korean power grid in Section 3 in terms of three aspects. Finally, we conclude the paper in Section 4.

Ⅱ. Topological Characteristics

In this section, we introduce the basic notion of graph theory and define several important topological characteristics. Also, we illustrate a method for obtaining these topological characteristics in the power grid.

2.1 Definition of Topological Characteristics
An undirected graph $G(V,E)$ is composed of V and E, which are sets of vertexes (nodes) and edges (links), respectively.[1] If an edge exists between vertexes i and j, the graph has the edge, i.e. $e_{i,j} = (i,j) \in E$. In power grids, a vertex refers to either a substation, transformer, or load, and an edge refers to a physical power cable directly connecting two nodes. Note that since the power grid is a type of undirected graph, all definitions in this paper refers to for the undirected graph.

The numbers of vertexes $N = |V|$ and edges $M = |E|$ are said to be the order and size of the

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1) In this paper, we interchangeably use the terms vertex and node, and edge and link.

graph, respectively. The nodal degree is defined as the number of edges which the node has. In undirected graphs, average nodal degree is defined as

$$<k> = \frac{2M}{N}. \tag{1}$$

The Pearson degree correlation $r \in [-1,1]$ shows the degree of the preference of nodes over other nodes which have similar node degrees. As $r \to -1$, nodes of dissimilar degree tend to be linked, while nodes of similar degree tend to be linked as $r \to 1$[13]. The mathematical formula for the Pearson degree correlation r is given as

$$r = \frac{\sum\limits_{(i,j) \in E} (k_i - \overline{k})(k_j - \overline{k})}{\sqrt{\sum\limits_{(i,j) \in E} (k_i - \overline{k})^2 (k_j - \overline{k})^2}}, \tag{2}$$

where k_i denotes the nodal degree for node i. $\overline{k}$ is defined as an average number of nodal degrees seen by a randomly chosen link. That is,

$$\overline{k} = \frac{1}{M} \sum\limits_{(i,j) \in E} \frac{(k_i + k_j)}{2}. \tag{3}$$

Note that $<k>$ and $\overline{k}$ are different variables. From (1) and (3), $<k>$ and $\overline{k}$ are average nodal degrees in a graph and in a randomly chosen edge. respectively.

Let $d_{i,j}$ denote the shortest path length between nodes i and j. To obtain $d_{i,j}$ the adjacency matrix J is needed, which is defined as

$$J(i,j) = \begin{cases} 1, \text{if } \exists \ (i,j) \in E, \\ 0, \text{otherwise}. \end{cases} \tag{4}$$

With J, we can obtain $d_{i,j}$ using Dijkstra's algorithm[14]. The average shortest path between two nodes is defined as

$$<l> = \frac{2}{N(N-1)} \sum\limits_{i > j} d_{i,j}. \tag{5}$$

$<l>$ is also known as the characteristic path length for the graph. Note that the characteristic path length for a random graph network[2) is obtained by

$$<l_{rand}> \approx \frac{\ln N}{\ln <k>}. \tag{6}$$

The network diameter d and network efficiency E are defined as

$$d = \max_{(i,j)} d_{i,j}, \tag{7}$$

and

$$E = \frac{1}{N(N-1)} \sum\limits_{i \neq j} \frac{1}{d_{i,j}}, \tag{8}$$

respectively. The network diameter illustrates the logical size of the network and the network efficiency describes the level of efficiency with which information spreads throughout the entire network[15].

The clustering coefficient C is a measure of the degree to which the network tends to cluster together[1]. Let C_i denote the clustering coefficient of node i, which is formally defined as

$$C_i = \frac{\lambda_G(i)}{\tau_G(i)}, \tag{9}$$

where $\lambda_G(i)$ and $\tau_G(i)$ denote the number of edges between node i's neighbors and the maximum number of links between node i's neighbors, respectively. Given k, $\tau_G(i) = k_i(k_i - 1)/2$ for undirected graphs. When C_i is 1, all neighbors of node i are connected to each other. The clustering coefficient is the average of C_i, i.e.

$$C = \frac{1}{N} \sum\limits_{i=1}^{N} C_i. \tag{10}$$

Note that the clustering coefficient for a random

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2) We use the random graph network proposed by Erdös-Rényi [18].

Table 1. Topological characteristics for US and Korea power grid networks. N: the number of nodes, M: the number of links, $<k>$: the average nodal degree, r: the Pearson degree correlation, $<l>$: the characteristic path length, d: the network diameter, E: the network's efficiency, C: the clustering coefficient.

	N	M	$<k>$	$\max k$	r	d	E	$<l>$	$<l_{rand}>$	C	C_{rand}
IEEE-30	30	41	2.73	7	-0.0875	6	0.3780	3.31	3.39	0.2348	0.09425
IEEE-118	118	179	3.03	9	-0.1528	14	0.2138	6.31	4.30	0.1651	0.02593
IEEE-300	300	409	2.73	11	-0.2240	24	0.1338	9.31	5.68	0.0856	0.00912
KO-765kV	5	4	1.6	2	-0.3333	4	0.6417	2	3.42	0	0
KO-345kV	93	118	2.54	6	-0.1817	17	0.2019	6.31	4.86	0.1559	0.02758
KO-154kV	860	1069	2.49	9	-0.1835	65	0.0516	26.31	7.41	0.1157	0.00289

graph network, i.e. C_{rand}, is equal to the probability of randomly selecting edges from all possible edges [1]. Therefore, it is obtained by

$$C_{rand} = \frac{2M}{N(N-1)}. \quad (11)$$

2.2 Matrix Representation for Power Grid

To obtain the above topological characteristics in the power grid, we use a similar method to that used in [16]. We use the incidence matrix A of the power grid rather than the admittance matrix[3] Y. For a power grid network of N order and M size, the matrix A is the $M \times N$ matrix. The mth row in A contains the information of link m, and the values of the mth row are $A(m,i)=1$, $A(m,i)=-1$, and $A(m,i)=0$, where $h \neq i$ or j when $m=(i,j)$.

With A, the $N \times N$ Laplacian matrix L is obtained as $L=A^T A$. Then, the matrix L is

$$L(i,j)=\begin{cases}-1, \text{if } \exists\, (i,j)\in E, \\ k_i, \text{ if } i=j, \\ 0, \text{ otherwise.}\end{cases}$$

The diagonal element for L is the nodal degree vector, i.e. $diag(L)=\vec{k}=[k_1,k_2,\cdots,k_N]$. With $\vec{k}$, we can obtain $<k>$, $\bar{k}$ and r. Using L, the adjacency matrix J can be obtained as

$$J=-L+\Lambda(diag(L)),$$

where $\Lambda(\cdot)$ is a function from a vector to a square matrix of which the diagonal elements are the vector[16].

Ⅲ. Korean Power Grid Analysis

Six power grid examples are analyzed in this work. The first three examples come from the reference transmission power grid in the US provided by IEEE[17] and the other three examples are derived from the Korean power grid. "IEEE-x" refers to the US reference grid with x number of nodes. "KO-ykV" refers to the Korea y kV transmission network.

3.1 Topological Characteristics

Table 1 shows the topological characteristics for the six power grid examples. $<l_{rand}>$ and C_{rand} are the characteristic path length and clustering coefficient, respectively, for a random graph network of which the order and degree are the same as those of the power grid in the same row.

In general, the topological characteristics for the both Korea and US grids are similar, except for KO-765kV. The topological analysis of the KO-765kV grid is not very significant because the KO-765kV grid is a very small, i.e. $N=5$ and $M=4$, and simple linear topology network. Therefore, the term "the five power grids" in the following text indicates that the KO-765kV grid is excluded from the six power grids. For each

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3) The admittance matrix contains impedance information of power cables as well as linking information. Since our analysis mainly focuses on topological characteristics for Korea power grid, we use the incidence matrix A rather than the admittance matrix Y.

topological characteristic, the analysis results show that

- Average nodal degrees $<k>$ for the five topologies are similar, i.e. $2.49 \leqq <k> \leqq 3.03$. That is, although the size of the network increases, $<k>$ does not increase. For the KO-154kV grid, the network is fully connected with $<k> = 2.49$, which is a very sparsely connected network.
- Maximum degrees of the Korean power grid networks are smaller than those of similar sized US grids. On the other hand, the network diameters d for Korean power grid networks are greater than those of similar sized US power grid networks. This is because the Korean power grid networks of 154 kV and 345 kV are ring-shaped topology networks.
- The Pearson degree correlations r for all power grids are negative. However, we cannot generalize that all power grids have negative Pearson degree correlations since some other power grids have a positive value[16]. For the five power grids, r is almost 0, which means that there is no specific preference for nodes to be linked with similar or dissimilar nodes.
- The network diameter d and network efficiency E tend to increase and decrease according to network size, respectively. This is because, as the number of nodes in a network increase, more time is required to spread the information.
- The characteristic path lengths of the five power grids $<l>$ are greater than those of random graph networks $<l_{rand}>$, except for "IEEE-30". This means that the average shortest path in the Korean power grid is greater than that in the random graph network with the same size. However, the difference is not severe, except for "KO-154kV".
- The clustering coefficients of the five power grids C are much greater than those of the random graph networks C_{rand}, while C of the Korean power grid is greater than that of the US reference grids. This means that the Korean power grids do not form a random graph

network; rather they form a small-world network. In Section 3.3, the small-world network characteristic of the Korean power grid is discussed in more detail.

To understand further the characteristics of the Korean power grid, a detailed analysis of the nodal degree and shortest path are followed.

3.2 Statistical Characteristics

The nodal degree represents the number of links of a node, and is an important characteristic for the power grid analysis. However, considering only the average and maximum nodal degrees are insufficient to understand the characteristic of power grids. Therefore, we investigate the empirical distributions for the nodal degree of the Korea power grid networks.

Note that the 765 kV transmission network is not included for both nodal degree and shortest path distributions because of its simple topology.

Fig. 2 shows the histogram and distribution fitting for the KO-345kV and KO-154kV power grid networks. Previous researches on nodal degree distributions for power grids have shown that the exponential distribution is the best fit for many power grids[9]. However, as shown in Fig. 2, the Korea power grid has the most frequent degree at $k = 2$, resulting in a high error with the exponential distribution.

To find the best fitting empirical probability density function (pdf), we use the maximum likelihood estimation. The candidate distributions are exponential, gamma, lognormal, Nakagami, Rayleigh, Rician, Weibull distributions, and so on. The best fitting distribution for both the 345 kV and 154 kV transmission networks, which has the maximum likelihood value, is the gamma distribution. The pdf for the gamma distribution is given as

$$f(x;a,b) = \frac{x^{a-1}e^{-\frac{x}{b}}}{b^a \Gamma(a)},$$

where a and b are the shape and scale for the
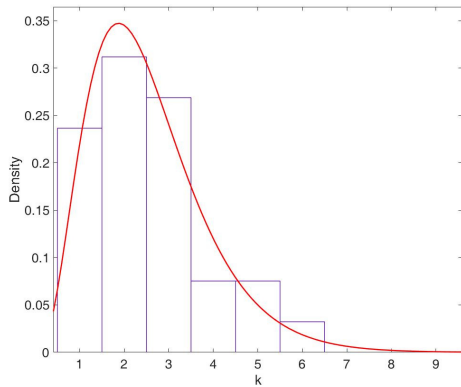
(a) KO-345kV power grid $<k> = 2.54$. The parameters for the gamma distribution are $a = 3.8222$ and $b = 0.6639$.
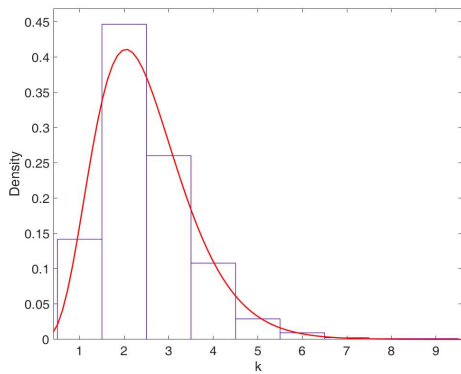
(b) KO-154kV power grid $<k> = 2.49$. The parameters for the gamma distribution are $a = 5.5855$ and $b = 0.4451$.

Fig. 2. Histograms for nodal degree in the Korean power grid networks and their distribution fitting. The gamma distribution (solid line) is their best fit.

gamma distribution, respectively, and $\Gamma(\bullet)$ is the gamma function. Using the maximum likelihood estimation, the parameters a and b are obtained for the KO-345kV and KO-154kV grids. They are $a = 3.8222$ and $b = 0.6639$ for 345 kV, and $a = 5.5855$ and $b = 0.4451$ for 154 kV. As shown in Fig. 2, the gamma distribution with the estimated parameters closely follows the original histogram.

We also examine the distribution fitting for the shortest path, which shows the network efficiency in more detail. We use the same approach to obtain the best fitting distribution, i.e., the maximum likelihood estimation. Fig. 3 shows histogram and the best

fitting distribution for the KO-345kV and KO-154kV power grids. The Rician distribution is the best fit for the shortest path histogram among the candidate distributions. The pdf of the Rician distribution is given as

$$f(x;\nu,\sigma) = \frac{x}{\sigma^2} exp\left(\frac{-(x^2+\nu^2)}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right),$$

where $I_0(\bullet)$ is the modified Bessel function of the first kind with order 0. The parameters obtained from the maximum likelihood estimation are $\nu = 5.4575$ and $\sigma = 3.6802$ for 345 kV, and $\nu = 20.485$ and $\sigma = 14.882$ for 154 kV.
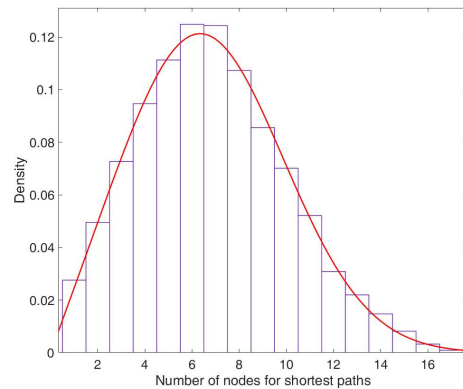
(a) KO-345kV power grid $<l> = 6.86$. The parameters for the Rician distribution are $\nu = 5.4575$ and $\sigma = 3.6802$.
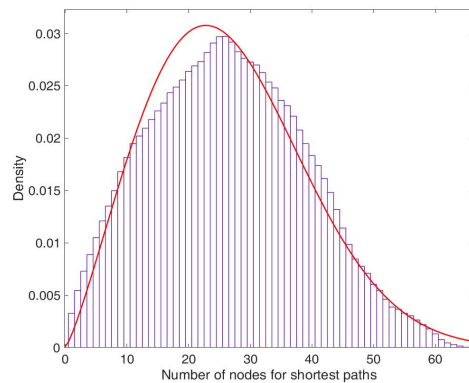
(b) KO-154kV power grid $<l> = 26.53$. The parameters for the Rician distribution are $\nu = 20.485$ and $\sigma = 14.882$.

Fig. 3. Histograms for the shortest path between two nodes in the Korean power grid networks and their distribution fitting. The Rician distribution (solid line) is their best fit.

3.3 Small-World Network Characteristic

The "small-world network" can be defined as *a type of mathematical graph in which most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of hops or steps.*[4] To generate a small-world network, each node initially has a link with its neighbors, i.e. ring topology, and randomly selects a node to connect with a node with a small probability[19].

While the power grid has been defined as a small-world network by the above definition, according to Watts-Strogatz[1], it is generally not a small-world network[9]. To determine whether or not the network is a small-world network, the Watts-Strogatz small-world model requires a condition, i.e. $N \gg <k> \gg \ln(N) \gg 1$.

The Korean power grid networks do not satisfy the middle inequality, i.e. $<k> \gg \ln(N)$, so they are not a small-world network according to Watts-Strogatz. In the case of the KO-154kV grid ($<k> = 2.49$), the maximum N needs to be about 12 in order to be a small-world network with the same $<k>$. In reality, since the KO-154kV grid has a much greater number of nodes, i.e. $N = 860$ than 12, it is a very sparse network compared to the small-world network. The KO-345kV grid has the same characteristic.

The Korean power grid is also not a random graph network because it has highly clustered networks that have much greater C than that of the random graph networks. Therefore, the Korean power grid is said to be highly clustered and has very sparse networks with reasonable characteristic path lengths.

Another efficient visualization method for the small-world network characteristic is the Kirk graph[20]. Fig. 4 shows the Kirk graph for the KO-345kV grid, small-world network, and random graph network. All three networks have the same order and size ($N = 93$ and $M = 118$). Again, it is confirmed that the Korean power grid is neither a small-world network nor a random graph network. Compared to the other two graphs, the KO-345kV
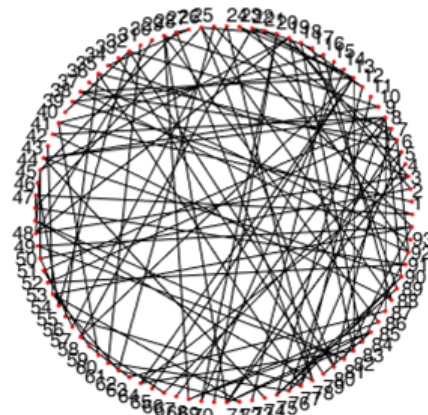
grid is the most clustered network.
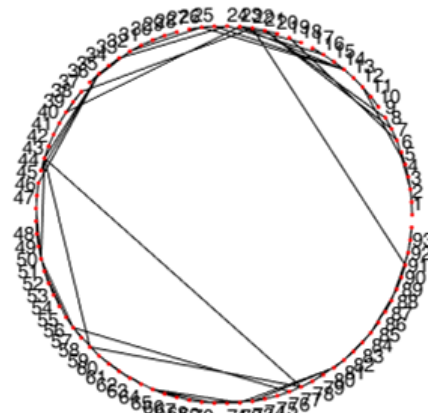
(a) KO-345kV power grid network
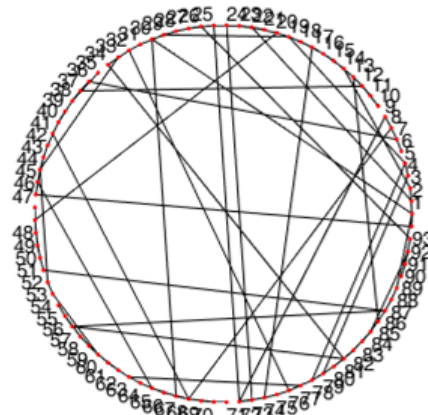
(b) Small-world network

(c) Random graph network

Fig. 4. Kirk graphs for the three networks. All the networks have 93 nodes and 118 links.

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4) https://en.wikipedia.org/wiki/Small-world_network

Ⅳ. Conclusion

One of the most "complex" networks among human-made networks is the power grid. Therefore, complex network analysis is required to examine the topological characteristics for a power grid. In this paper, we analyzed the Korean power grid through complex network analysis. The Korean power grid has many similarities to the US reference power grid, e.g. average nodal degree and Pearson correlation coefficient. However, some characteristics such as nodal degree distribution and network efficiency differ between the Korea and the US power grids. This is because the Korean power grid has a ring-shaped topology covering the capital area. It has also shown that the Korean power grid is neither a small-world network nor a random graph network. Its networks are highly clustered and sparsely connected with reasonable characteristic path lengths. Using this complex network analysis on the power grid, the grid vulnerability can be analyzed, and it is left as a future work.

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